Question			Answer	Marks	Guidance	
1			$y^2 + 2x \ln y = x^2$			
			$1^2 + 2\times1\times\ln1 = 1^2$ so (1, 1) lies on the curve.	B1	clear evidence of verification needed	at least "1 + 0 = 1"
				M1	d/dx (y^2) = 2ydy/dx	must be correct
			$2y\frac{dy}{dx}+2\ln y+2x.\frac{1}{y}.\frac{dy}{dx}=2x$	M1 A1 cao	d/dx (2x ln y) = 2ln y + 2x/y dy/dx	must be correct condone dy/dx = … unless pursued
			[$\Rightarrow \frac{dy}{dx}=\frac{2x-2\ln y}{2y+2x/y}$]			
			when $x = 1$, $y = 1$, $\frac{dy}{dx}=\frac{2-2\ln1}{2+2}$	M1	substituting both x = 1 and y = 1 into their dy/dx or their equation in x, y and dy/dx	$2\frac{dy}{dx}+2\ln1+2\frac{dy}{dx}=2$
			= ½	A1cao	not from wrong working	
				[6]		

2	**(i)**		$2x + 4y\frac{dy}{dx} = 4$	M1	$4y\frac{dy}{dx}$	Rearranging for y and differentiating explicitly is M0
				A1	correct equation	Ignore superfluous $dy/dx = \ldots$ unless used subsequently
			$\Rightarrow \quad \frac{dy}{dx} = \frac{4-2x}{4y}$	A1	o.e., but mark final answer	
				[3]		
2	**(ii)**		$\frac{dy}{dx} = 0 \Rightarrow x = 2$	B1dep	dep correct derivative	
			$\Rightarrow \quad 4 + 2y^2 = 8 \Rightarrow y^2 = 2,\ y = \sqrt{2}$ or $-\sqrt{2}$	B1B1	$\sqrt{2}, -\sqrt{2}$	can isw, penalise inexact answers of ±1.41 or better once only
				[3]		−1 for extra solutions found from using $y = 0$

3			$y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right) = \frac{1}{2}(\ln(2x-1) - \ln(2x+1))$	M1	use of $\ln(a/b) = \ln a - \ln b$
				M1	use of $\ln\sqrt{c} = ½ \ln c$
			$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}\left(\frac{2}{2x-1} - \frac{2}{2x+1}\right)$	A1	o.e.; correct expression (if this line of working is missing, M1M1A0A0)
			$= \frac{1}{2x-1} - \frac{1}{2x+1}$ *	A1	**NB AG**
				[4]	for alternative methods, see additional solutions

4	**(i)**		$x^3 + y^3 = 3xy$ $\Rightarrow \quad 3x^2 + 3y^2(\mathrm{d}y/\mathrm{d}x) = 3x(\mathrm{d}y/\mathrm{d}x) + 3y$ $\Rightarrow \quad (3y^2 - 3x)(\mathrm{d}y/\mathrm{d}x) = 3y - 3x^2$ $\Rightarrow \quad \mathrm{d}y/\mathrm{d}x = (3y - 3x^2)/(3y^2 - 3x)$ $= (y - x^2)/(y^2 - x)$*	B1B1 M1 A1cao [4]	LHS, RHS Condone 3*x*d*y*/d*x*+*y* (i.e.with missing bracket) if recovered thereafter collecting terms in d*y*/d*x* and factorising NB **AG**	or equivalent if re-arranged. ft correct algebra on incorrect expressions with two d*y*/d*x* terms Ignore starting with 'd*y*/d*x* = …' unless pursued
	(ii)		TP when $y - x^2 = 0$ $\Rightarrow \quad y = x^2$ $\Rightarrow \quad x^3 + x^6 = 3x.x^2$ $\Rightarrow \quad x^6 = 2x^3$ $\Rightarrow \quad x^3 = 2$ (or $x = 0$) $\Rightarrow \quad x = \sqrt[3]{2}$	 M1 M1 A1 A1cao [4]	 or $x = \sqrt{y}$ substituting for *y* in implicit eqn (allow one slip, e.g. x^5) o.e. (so must be exact	 or *x* for *y* (i.e. $y^{3/2} + y^3 = 3y^{1/2}y$ o.e.) or $y^{3/2} = 2$ $x = 1.2599…$ is A0 (but can isw $x = \sqrt[3]{2}$)

5			$\mathrm{e}^{2y} = 5 - \mathrm{e}^{-x}$ $\Rightarrow \quad 2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}$ $\Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{-x}}{2\mathrm{e}^{2y}}$ At (0, ln2) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{0}}{2\mathrm{e}^{2\ln 2}}$ $= \frac{1}{8}$	B1 B1 M1dep A1cao **[4]**	$2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ldots$ $= \mathrm{e}^{-x}$ substituting *x* = 0, *y* = ln 2 into their d*y*/d*x* dep 1st B1 – allow one slip	or $y = \ln\sqrt{(5 - \mathrm{e}^{-x})}$ o.e (e.g.½ $\ln(5 - \mathrm{e}^{-x})$)B1 $\Rightarrow \mathrm{d}y/\mathrm{d}x = \mathrm{e}^{-x}/[2(5 - \mathrm{e}^{-x})]$ o.e. B1 (but must be correct) or substituting *x* = 0 into their correct d*y*/d*x*

6 $(x+y)^2 = 4x$ $\Rightarrow$ $2(x+y)(1+\frac{dy}{dx}) = 4$ $\Rightarrow$ $1+\frac{dy}{dx} = \frac{4}{2(x+y)} = \frac{2}{x+y}$ $\Rightarrow$ $\frac{dy}{dx} = \frac{2}{x+y} - 1$ *	M1 A1 A1	Implicit differentiation of LHS correct expression = 4 www (**AG**)	Award no marks for solving for y and attempting to differentiate allow one error but must include dy/dx ignore superfluous dy/dx = … for M1, and for both A1s if not pursued condone missing brackets A0 if missing brackets in earlier working
or $x^2+2xy+y^2 = 4x$ $\Rightarrow$ $2x+2x\frac{dy}{dx}+2y+2y\frac{dy}{dx} = 4$ $\Rightarrow$ $\frac{dy}{dx}(2x+2y) = 4-2x-2y$ $\Rightarrow$ $\frac{dy}{dx} = \frac{4}{2x+2y} - 1 = \frac{2}{x+y} - 1$ *	M1dep A1 A1	Implicit differentiation of LHS dep correct expansion correct expression = 4 (oe after re-arrangement) www (**AG**)	allow 1 error provided 2xdy/dx and 2ydy/dx are correct, but must expand $(x + y)^2$ correctly for M1 (so $x^2 + y^2 = 4x$ is M0) ignore superfluous dy/dx = … for M1, and for both A1s if not pursued A0 if missing brackets in earlier working
When $x = 1$, $y = 1$, $\frac{dy}{dx} = \frac{2}{1+1} - 1 = 0$ *	B1 [4]	(**AG**) oe (e.g. from $x + y = 2$)	or e.g $2/(x + y) - 1 = 0 \Rightarrow x + y = 2, \Rightarrow 4 = 4x, \Rightarrow x = 1, y = 1$ (oe)

7(i) sin(π/3) + cos(π/6) = √3/2 + √3/2 = √3	B1 [1]	must be exact, must show working	Not just sin(π/3) + cos(π/6) = √3, if substituting for *y* and solving for *x* (or vv) must evaluate sin π/3 e.g. not arcos(√3 – sin π/3)
(ii) $2\cos 2x - \sin y\frac{dy}{dx} = 0$ ⇒ $2\cos 2x = \sin y\frac{dy}{dx}$ ⇒ $\frac{dy}{dx} = \frac{2\cos 2x}{\sin y}$ When *x* = π/6, *y* = π/6 ⇒ $\frac{dy}{dx} = \frac{2\cos \pi/3}{\sin \pi/6} = 2$	M1 A1 A1cao M1dep A1 [5]	Implicit differentiation correct expression substituting dep 1st M1 www	allow one error, but must have (±) sin *y* d*y*/d*x*. Ignore d*y*/d*x* = … unless pursued. 2cos 2*x* d*x* – sin *y* d*y* = 0 is M1A1 (could differentiate wrt *y*, get d*x*/d*y*, etc.) $\frac{-2\cos 2x}{-\sin y}$ is A0 or 30°

8(i) $\frac{dy}{dx} = \frac{1}{3}(1+3x^2)^{-2/3}.6x$ $= 2x(1+3x^2)^{-2/3}$	M1 B1 A1 [3]	chain rule 1/3 $u^{-2/3}$ or $\frac{1}{3}(1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer
(ii) $3y^2\frac{dy}{dx} = 6x$ ⇒ d*y*/d*x* = 6*x*/3*y*² $= \frac{2x}{(1+3x^2)^{2/3}} = 2x(1+3x^2)^{-2/3}$	M1 A1 A1 E1 [4]	$3y^2\frac{dy}{dx}$ = 6*x* if deriving $2x(1+3x^2)^{-2/3}$, needs a step of working